5.1 The Product Rule and Power Rules for Exponents

Base	Exponent/Power

EXAMPLE 1 Using Exponents

Write $3 \cdot 3 \cdot 3 \cdot 3$ in exponential form and evaluate.

EXAMPLE 2 Evaluating Exponential Expressions

Evaluate. Name the base and the exponent.

(a) 5^4 **(b)** -5^4 **(c)** $(-5)^4$

(d) $(-5)^3$ **(e)** $-(-5)^3$

Product rule for exponents

EXAMPLE 3 Using the Product Rule

Use the product rule for exponents to find each product if possible.

(a) $6^3 \cdot 6^5$ **(b)** $(-4)^7(-4)^2$ **(c)** $x^2 \cdot x$ **(d)** $m^4m^3m^5$

(e) $2^3 \cdot 3^2$ **(f)** $2^3 + 2^4$ **(g)** $(2x^3)(3x^7)$

(h) $(m+n)^2(m+n)^3$

Power rules for exponents

a) b) c)

Example 4: Use the Power Rules for exponents to simplify each expression.

a) $(2^5)^3$

b) $(5^7)^2$

c) $(x^2)^5$

d) $(3xy)^2$

e) $5(4pq)^2$

f) $3(2m^2p^3)^4$

g) $(-5^6)^3$

h) $\left(\frac{2}{3}\right)^5$

i) $\left(\frac{m}{n}\right)^3$

j) $\left(\frac{1}{5}\right)^4$

Example 5: Simplify by using a combination of rules.

(a) $\left(\frac{2}{3}\right)^2 \cdot 2^3$

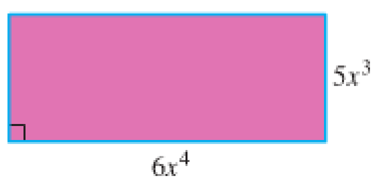
(b) $(5x)^3(5x)^4$

(c) $(2x^2y^3)^4(3xy^2)^3$

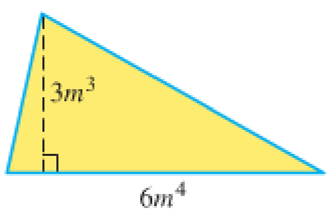
(d) $(-x^3y)^2(-x^5y^4)^3$

Example 6: Using Area Formulas. Find an expression that represents the area in each figure.

a)



b)



5.2 Integer Exponents and the Quotient Rules

OBJECTIVES

1. Use 0 as an exponent.
2. Use negative numbers as exponents.
3. Use the quotient rule for exponents.
4. Use combinations of rules.

Zero exponent

EXAMPLE 1 Using Zero Exponents

Evaluate.

(a) 60^0 (b) $(-60)^0$

(c) -60^0 (d) y^0

(e) $6y^0$ (f) $(6y)^0$

(g) $8^0 + 11^0$ (h) $-8^0 - 11^0$

Negative exponents

EXAMPLE 2 Using Negative Exponents

Simplify by writing with positive exponents. Assume that all variables represent nonzero real numbers.

(a) 3^{-2} (b) 5^{-3} (c) $\left(\frac{1}{2}\right)^{-3}$ (d) $\left(\frac{2}{5}\right)^{-4}$

(e) $\left(\frac{4}{3}\right)^{-5}$ (f) $4^{-1} - 2^{-1}$ (g) p^{-2} (h) $\frac{1}{x^{-4}}$ (i) x^3y^{-4}

Changing from Negative to Positive Exponents

For any nonzero numbers a and b and any integers m and n, the following are true.

$$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m} \quad \text{and} \quad \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$$

Examples: $\frac{3^{-5}}{2^{-4}} = \frac{2^4}{3^5}$ and $\left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{4}\right)^3$

EXAMPLE 3 Changing from Negative to Positive Exponents

Simplify by writing with positive exponents. Assume that all variables represent nonzero real numbers.

(a) $\frac{4^{-2}}{5^{-3}}$

(b) $\frac{m^{-5}}{p^{-1}}$

(c) $\frac{a^{-2}b}{3d^{-3}}$

(d) $\left(\frac{x}{2y}\right)^{-4}$

Quotient rule for exponents

EXAMPLE 4 Using the Quotient Rule

Simplify by writing with positive exponents. Assume that all variables represent nonzero real numbers.

(a) $\dfrac{5^8}{5^6}$

(b) $\dfrac{4^2}{4^9}$

(c) $\dfrac{5^{-3}}{5^{-7}}$

(d) $\dfrac{q^5}{q^{-3}}$

(e) $\dfrac{3^2x^5}{3^4x^3}$

(f) $\dfrac{(m+n)^{-2}}{(m+n)^{-4}}$

(g) $\dfrac{7x^{-3}y^2}{2^{-1}x^2y^{-5}}$

EXAMPLE 5 Using Combinations of Rules

Simplify. Assume that all variables represent nonzero real numbers.

(a) $\dfrac{(4^2)^3}{4^5}$

(b) $(2x)^3(2x)^2$

(c) $\left(\dfrac{2x^3}{5}\right)^{-4}$

(d) $\left(\dfrac{3x^{-2}}{4^{-1}y^3}\right)^{-3}$

(e) $\dfrac{(4m)^{-3}}{(3m)^{-4}}$

5.3 An Application of Exponents: Scientific Notation

OBJECTIVES

1. Express numbers in scientific notation.
2. Convert numbers in scientific notation to numbers without exponents.
3. Use scientific notation in calculations.

Scientific Notation

EXAMPLE 1 Using Scientific Notation

Write each number in scientific notation.

(a) 93,000,000

(b) 63,200,000,000

(c) 0.00462

(d) -0.0000762

EXAMPLE 2 Writing Numbers without Exponents

Write each number without exponents.

(a) 6.2×10^3

(b) 4.283×10^6

(c) 7.04×10^{-3}

EXAMPLE 3 Multiplying and Dividing with Scientific Notation

Perform each calculation.

(a) $(7 \times 10^3)(5 \times 10^4)$

(b) $\dfrac{4 \times 10^{-5}}{2 \times 10^3}$

(c) $(3 \times 10^4)(8 \times 10^7)$

(d) $\dfrac{6 \times 10^{-2}}{2 \times 10^8}$

EXAMPLE 4 Using Scientific Notation to Solve an Application

A *nanometer* is a very small unit of measure that is equivalent to about 0.00000003937 in. About how much would 700,000 nanometers measure in inches? (*Source: World Almanac and Book of Facts*.)

EXAMPLE 5 Using Scientific Notation to Solve an Application

In 2008, the national debt was $\$1.0025 \times 10^{13}$ (which is more than \$10 trillion). The population of the United States was approximately 304 million that year. About how much would each person have had to contribute in order to pay off the national debt? (*Source*: Bureau of Public Land; U.S. Census Bureau.)

5.4 Adding and Subtracting Polynomials; Graphing Simple Polynomials

OBJECTIVES

1. Identify terms and coefficients.
2. Add like terms.
3. Know the vocabulary for polynomials.
4. Evaluate polynomials.
5. Add and subtract polynomials.
6. Graph equations defined by polynomials of degree 2.

Terms	Numerical coefficient

EXAMPLE 1 Identifying Coefficients

Name the coefficient of each term in these expressions.

(a) $x - 6x^4$

(b) $5 - v^3$

EXAMPLE 2 Adding Like Terms

Simplify by adding like terms.

(a) $-4x^3 + 6x^3$

(b) $9x^6 - 14x^6 + x^6$

(c) $12m^2 + 5m + 4m^2$

(d) $3x^2y + 4x^2y - x^2y$

Standard form

Degree of a term

Degree of a polynomial

Monomial

Binomial

Trinomial

Term	*Degree*
$3x^4$	
$5x$, or $5x^1$	
-7, or $-7x^0$	
$2x^2y$, or $2x^2y^1$	

Polynomial	*Degree*
$3x^4 - 5x^2 + 6$	
$5x + 7$	
$x^2y + xy - 5y^2$	
$x^5 + 3x^6$	

EXAMPLE 3 Classifying Polynomials

For each polynomial, first simplify, if possible. Then give the degree and tell whether the polynomial is a *monomial,* a *binomial,* a *trinomial,* or *none of these.*

(a) $2x^3 + 5$ **(b)** $4xy - 5xy + 2xy$

EXAMPLE 4 Evaluating a Polynomial

Find the value of $3x^4 + 5x^3 - 4x - 4$ for **(a)** $x = -2$ and **(b)** $x = 3$.

Adding polynomials

EXAMPLE 5 Adding Polynomials Vertically

$(\boldsymbol{a})\ Add\text{: } (6x^3 - 4x^2 + 3) + (-2x^3 + 7x^2 - 5).$

EXAMPLE 6 Adding Polynomials Horizontally

$(\boldsymbol{a})\ Add\text{: } (6x^3 - 4x^2 + 3) + (-2x^3 + 7x^2 - 5).$

Subtracting polynomials

EXAMPLE 7 Subtracting Polynomials Horizontally

(a) Perform the subtraction $(5x - 2) - (3x - 8)$.

$(\boldsymbol{b})\ Subtract\text{: } (6x^3 - 4x^2 + 2) - (11x^3 + 2x^2 - 8).$

EXAMPLE 8 Subtracting Polynomials Vertically

Subtract by columns to find

$$(14y^3 - 6y^2 + 2y - 5) - (2y^3 - 7y^2 - 4y + 6).$$

EXAMPLE 9 Adding and Subtracting Polynomials with More Than One Variable

Add or subtract as indicated.

(a) $(4a + 2ab - b) + (3a - ab + b)$

(b) $(2x^2y + 3xy + y^2) - (3x^2y - xy - 2y^2)$

(c) $(8a^3 - 2a^2 + 3) + (-2a^3 + 6a - 2)$.

5.5 Multiplying Polynomials

OBJECTIVES

1. Multiply a monomial and a polynomial.
2. Multiply two polynomials.
3. Multiply binomials by the FOIL method.

EXAMPLE 1 Multiplying Monomials and Polynomials

Find each product.

(a) $4x^2(3x + 5)$

(b) $-8m^3(4m^3 + 3m^2 + 2m - 1)$

EXAMPLE 2 Multiplying Two Polynomials

a) Multiply $(m^2 + 5)(4m^3 - 2m^2 + 4m)$.

b) Multiply $(x^3 + 2x^2 + 4x + 1)(3x + 5)$.

EXAMPLE 4 Multiplying Polynomials with Fractional Coefficients

Find the product of $4m^3 - 2m^2 + 4m$ and $\frac{1}{2}m^2 + \frac{5}{2}$.

FOIL

EXAMPLE 5 Using the FOIL Method

Use the FOIL method to find the product $(x + 8)(x - 6)$.

EXAMPLE 6 Using the FOIL Method

Multiply $(9x - 2)(3y + 1)$.

EXAMPLE 7 Using the FOIL Method

Find each product.

(a) $(2k + 5y)(k + 3y)$

(c) $2x^2(x - 3)(3x + 4)$

5.6 Special Products

OBJECTIVES

1. Square binomials.
2. Find the product of the sum and difference of two terms.
3. Find greater powers of binomials.

EXAMPLE 1 Squaring a Binomial

Find $(m + 3)^2$.

EXAMPLE 2 Squaring Binomials

Find each binomial square and simplify.

(a) $(5z - 1)^2$

(b) $(3b + 5r)^2$

(c) $(2a - 9x)^2$

(d) $\left(4m + \frac{1}{2}\right)^2$

(e) $x(4x - 3)^2$

Conjugates:

Example 3: Find the product of each set of conjugates.

(a) $(x + 4)(x - 4)$

(b) $(x + 10)(x - 10)$

(c) $(x - 5)(x + 5)$

(d) $\left(\frac{2}{3} - w\right)\left(\frac{2}{3} + w\right)$

(e) $(5m - 3)(5m + 3)$

(f) $(4x - 6y)(4x + 6y)$

(g) $\left(z - \frac{1}{4}\right)\left(z + \frac{1}{4}\right)$

EXAMPLE 5 Finding Greater Powers of Binomials

Find each product.

(a) $(x + 5)^3$

(c) $-2r(r + 2)^3$